

This is the case at the minimum or maximum of a "parabolic" band where the dependence of E on k can be well approximated by

$$E(k) = E_0 \pm \frac{\hbar^2}{2m^*} (k_x^2 + k_y^2 + k_z^2) . \quad (9.10)$$

In the vicinity of such a critical point, the so-called *effective-mass approximation* is particularly useful, because here m^* is a constant. On moving away from this point along the band, the deviation of the $E(k)$ surface from the parabolic form (9.10) means that m^* depends on k .

Figure 9.2 shows two one-dimensional bands $E(k)$ with strong (a) and weak (b) curvature at the top and bottom of the bands. The effective mass is correspondingly small (a) and large (b) at these points. At the Brillouin zone boundary (top of the band), where the curvature is negative, the effective mass is also negative. Here it can clearly be seen that the effective-mass concept conveniently describes the effect of the periodic potential in terms of the k -dependent quantity m^* .

It should be remarked with respect to the above description that one can still retain all the results derived in Chap. 6 for the free electron gas in metals. This is true so long as we consider only electrons in bands $E(k)$ that can be described by the parabolic approximation (9.10). Near to the band extrema the free electron mass m is simply replaced by the effective mass m^* which is a constant.

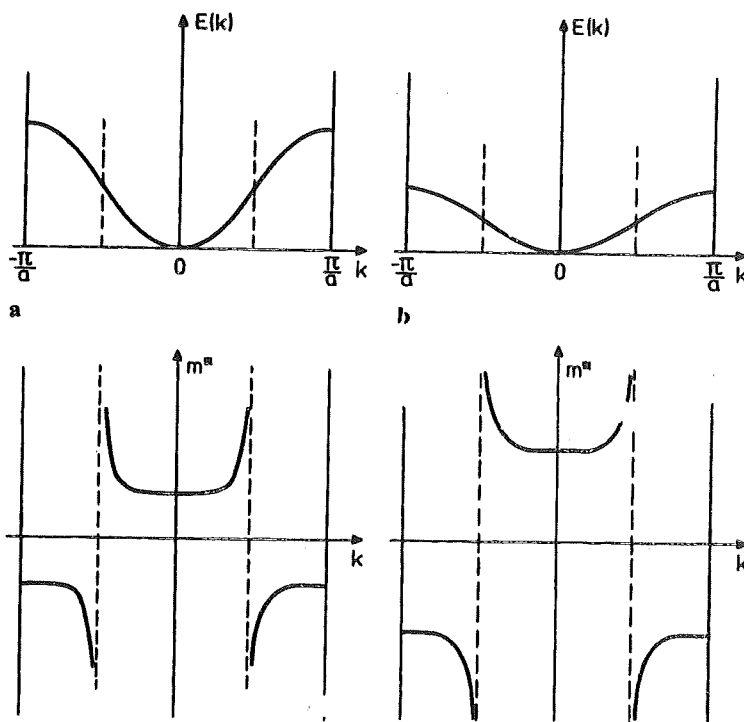


Fig. 9.2. Schematic behavior of the effective mass $m^*(k)$ for a one-dimensional bandstructure $E(k)$: a for strong curvature of the bands, i.e., small effective masses; b for weak curvature, i.e., large effective masses. The dashed lines denote the points of inflection of $E(k)$

CHE 520, AVILIMOVY-LEKCIJE, 1978, 1979, 1980, 1981, 1982, 1983, 1984, 1985, 1986, 1987, 1988, 1989, 1990, 1991, 1992, 1993, 1994, 1995, 1996, 1997, 1998, 1999, 2000, 2001, 2002, 2003, 2004, 2005, 2006, 2007, 2008, 2009, 2010, 2011, 2012, 2013, 2014, 2015, 2016, 2017, 2018, 2019, 2020, 2021, 2022, 2023, 2024, 2025, 2026, 2027, 2028, 2029, 2030, 2031, 2032, 2033, 2034, 2035, 2036, 2037, 2038, 2039, 2040, 2041, 2042, 2043, 2044, 2045, 2046, 2047, 2048, 2049, 2050, 2051, 2052, 2053, 2054, 2055, 2056, 2057, 2058, 2059, 2060, 2061, 2062, 2063, 2064, 2065, 2066, 2067, 2068, 2069, 2070, 2071, 2072, 2073, 2074, 2075, 2076, 2077, 2078, 2079, 2080, 2081, 2082, 2083, 2084, 2085, 2086, 2087, 2088, 2089, 2090, 2091, 2092, 2093, 2094, 2095, 2096, 2097, 2098, 2099, 2100, 2101, 2102, 2103, 2104, 2105, 2106, 2107, 2108, 2109, 2110, 2111, 2112, 2113, 2114, 2115, 2116, 2117, 2118, 2119, 2120, 2121, 2122, 2123, 2124, 2125, 2126, 2127, 2128, 2129, 2130, 2131, 2132, 2133, 2134, 2135, 2136, 2137, 2138, 2139, 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$$EB > E_g$$

Gap piccola

Gap ampia $EB < E_g$

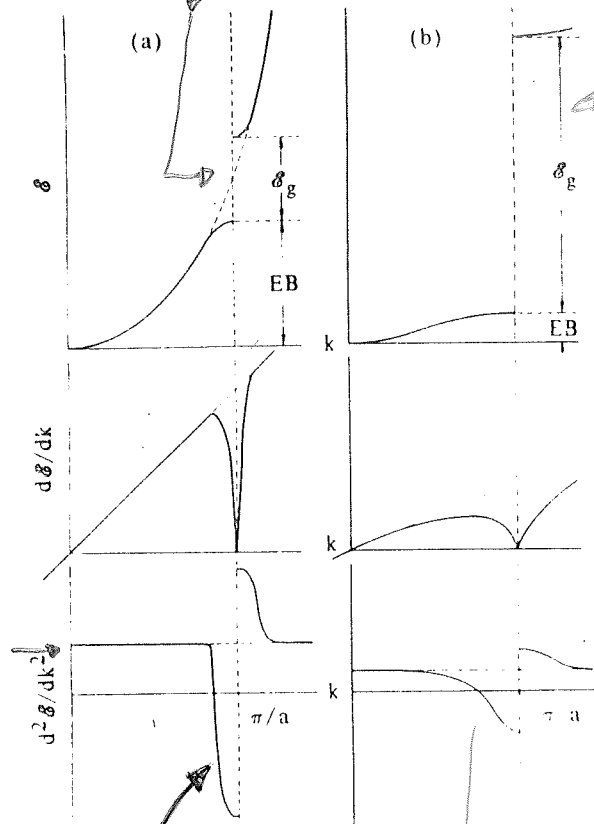


Fig. 10-3 Plots of δ and various derivatives with respect to k showing what happens near $k = \pi/a$. (a) is for narrow band gaps as well as for wide gaps. (b) is the same but emphasizes what the situation for wide band gaps, that is, for $E_g \gg EB$.

$$\frac{1}{m^*} \sim \frac{1}{m}$$

$$\textcircled{\times} \text{ Se } E \sim \frac{\hbar^2 k^2}{2m} \rightarrow \frac{\partial^2 E}{\partial k^2} = \frac{\hbar^2}{m}$$

$$\frac{1}{m^*} \sim \frac{1}{m}$$

$$\frac{1}{m^*} < 0$$

$$E \neq \frac{\hbar^2 k^2}{2m} \rightarrow \frac{1}{m^*} \neq \frac{1}{m}$$

caso tipico di elettroni di valenza molto localizzati \rightarrow semicond.

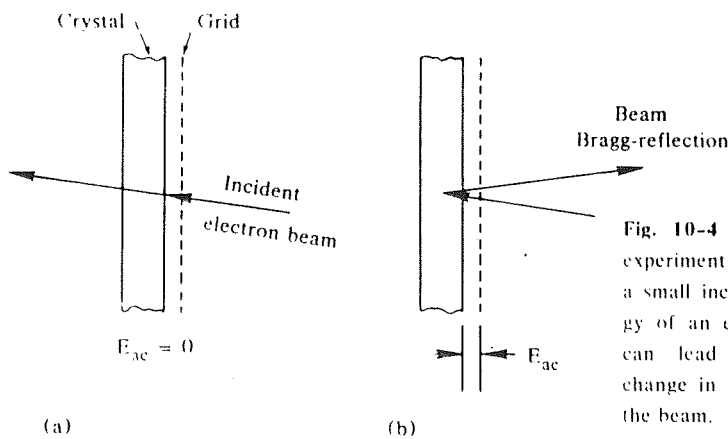


Fig. 10-4 A gedanken experiment showing how a small increase in energy of an electron beam can lead to a large change in momentum of the beam.

$$2 d \sin \theta = m \lambda$$

Se $E(\text{elettr.})$ aumenta \rightarrow riflessione alle Bragg.
 \rightarrow inversione del momento $\rightarrow m^* < 0$
 $\vec{F} = -m^* \vec{a}$